A Maximum *a Posteriori* Sound Source Localization in Reverberant and Noisy Conditions

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Abstract

In this paper, a maximum *a posteriori* sound source localization (MAP-SSL) algorithm is proposed for reverberant and noisy conditions. It is derived by incorporating a sparse prior on the source location into the existing maximum likelihood sound source localization (ML-SSL) framework. The criterion in deriving the proposed MAP-SSL algorithm is similar to the criterion used in deriving the existing ML-SSL framework, except that in the proposed criterion, a sparse source prior is added. The source prior that enforces a sparse solution plays a key role in improving the SSL performance. The experimental results show the proposed MAP-SSL algorithm outperforms two popular algorithms based on the ML-SSL framework.

Index Terms: Sound source localization, reverberation, a sparse source prior .

1. Introduction

A sound source localization (SSL) algorithm estimates the location and direction of the sound source based on the received signals at the microphone array. The estimation is often considered in reverberant and noisy environment, and under such conditions, many SSL algorithms have been proposed [1, 2, 3, 4]. Recently, the maximum likelihood sound source localization (ML-SSL) framework proposed by Zhang *et al.* [1] has shown to be effective in both reverberant and noisy conditions. The ML-SSL framework is also closely related to the steered response power-phase transform (SRP-PHAT) algorithm [5]. The ML-SSL framework searches all possible finite source directions and finds the direction of source that maximizes the cost function proposed in [1].

This paper proposes an MAP-SSL algorithm for the SSL in reverberant and noisy conditions. The proposed MAP-SSL algorithm is based on maximizing the posteriori of the source. The proposed MAP-SSL algorithm is derived using the sparse representation. In [6], the sparse signal representation was used without considering reverberation for both narrow-band signal and wide-band signal localization. The sparse signal representation can be written as follows: $\mathbf{x} = \mathbf{A}\mathbf{s} + \mathbf{v}$ where $\mathbf{x} \in \mathbb{C}^M$, $\mathbf{A} \in \mathbb{C}^{M\times N}$, $\mathbf{s} \in \mathbb{C}^N$ and $\mathbf{v} \in \mathbb{C}^M$ are a vector representing targets, a matrix describing possible over-complete basis, a source vector to be learned and additive Gaussian noise, respectively. When only a single source is active, the source vector $\mathbf{s}$ must be estimated with most of its entries set to zeros while minimizing the square error between $\mathbf{A}\mathbf{s}$ and $\mathbf{x}$. Various algorithms have been proposed to obtain $\mathbf{s}$ using the sparse signal representation, and a recent algorithm that has shown great promise is the sparse Bayesian learning (SBL) [7]. This algorithm obtains an estimate of the source vector $\mathbf{s}$ that minimizes the l_0-norm by estimating the variance of the source posterior probability such that the variance of the source vector entry with value near zero should be near zero. This SBL is applied to solve our criterion in reverberant and noisy conditions. When a source is active, the proposed MAP-SSL algorithm estimates a sparse solution for the location of source, and the sparse source prior term in the proposed MAP-SSL algorithm plays a key role in improving the source localization performance over other algorithms such as the various variants of ML-SSL framework.

The paper is organized as follows. Section 2 formulates the problem and takes assumptions. Section 3 proposes a maximum *a posterori* sound source localization algorithm. Section 4 gives the performance evaluation of the proposed MAP-SSL algorithm. Finally Section 5 concludes the paper.

2. Problem formulation and assumptions

We consider the SSL in reverberant and noisy conditions. In our formulation, assuming far field [5] i. e. that a source is very far from the microphones, the set of all possible finite source directions can be given as a function of the all possible finite integer time delays,

$$\theta = \arcsin(\frac{\tau C}{f_s d}) \tag{1}$$

where C, f_s and d are the propagation velocity of sound, the sampling rate and the distance between two microphones. Here, $\tau \in [-\tau^{max}, \tau^{max}]$ is the time delay with the maximum possible time delay τ^{max}, given C, f_s, and d. The range of θ is from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ as a unit of radian. Let a direction of an active source such that $s_a[n]$ denotes active source from $\frac{\pi(a-1)}{N-1} - \frac{\pi}{2}$ direction. When a microphone array is composed of M microphones with an arbitrary layout for placement, the signal model in time domain is given as,

$$x_m[n] \quad = \quad \sum_{i=1}^{N} h_{mi}[n] * s_i[n] + v_m[n] \tag{2}$$

where $*$ denotes convolution, $i = 1, .., N$ is the index of source, and $m = 1, ..., M$ is the index of microphone. Here, $x_m[n]$, $s_i[n]$, $h_{mi}[n]$ and $v_m[n]$ are the received signal at the mth microphone, the ith direction source, the room impulse response from the ith direction source to the mth microphone and the additive Gaussian noise at the mth microphone. Note that the summation term includes only one active source. Assuming that room impulse response $h_{mi}[n]$ does not change with time, the Fourier transform of (2) is given as

$$X_m(\omega) \quad = \quad \sum_{i=1}^{N} H_{mi}(\omega) S_i(\omega) + V_m(\omega) \tag{3}$$

where $X_m(\omega)$, $S_i(\omega)$, $H_{mi}(\omega)$ and $V_m(\omega)$ are the Fourier transforms of $x_m[n]$, $s_i[n]$, $h_{mi}[n]$ and $v_m[n]$, respectively. Here, $H_{mi}(\omega)$ can be decomposed into direct path and reverberant path signals [1, 3], as

$$H_{mi}(\omega) = a_{mi}(\omega)e^{-j\omega\tau_{mi}} + R_{mi}(\omega) \quad (4)$$

Therefore,

$$\begin{aligned} X_m(\omega) &= \sum_{i=1}^{N} [a_{mi}(\omega)e^{-j\omega\tau_{mi}} + R_{mi}(\omega)]S_i(\omega) \\ &+ V_m(\omega) \end{aligned} \quad (5)$$

where $a_{mi}(\omega)e^{-j\omega\tau_{mi}}$ and $R_{mi}(\omega)$ are the direct path and reverberant path signal from the i directional source to the mth microphone. Here the time delay of the direct path is denoted as τ_{mi}. If we consider the 1st microphone as the reference microphone, the τ_{mi} can be described as the relative time delay from the 1st microphone to the mth microphone as

$$\tau'_{mi} = \tau_{1i} - \tau_{mi}. \quad (6)$$

The vector-matrix form of (5) is given as,

$$\mathbf{x}(\omega) = \mathbf{A}(\omega)\mathbf{s}(\omega) + \mathbf{R}(\omega)\mathbf{s}(\omega) + \mathbf{v}(\omega) \quad (7)$$

where

$$\begin{aligned} \mathbf{x}(\omega) &= [X_1(\omega), ..., X_M(\omega)]^T (\in \mathbb{C}^M), \\ \mathbf{A}(\omega) &= [\mathbf{a}_1(\omega), ..., \mathbf{a}_N(\omega)] (\in \mathbb{C}^{M\times N}), \\ \mathbf{a}_i(\omega) &= [a_{1i}(\omega), a_{2i}(\omega)e^{-j\omega\tau'_{2i}}, ..., a_{Mi}(\omega)e^{-j\omega\tau'_{Mi}}]^T (\in \mathbb{C}^M), \\ \mathbf{s}(\omega) &= [S_1(\omega), ..., S_N(\omega)]^T (\in \mathbb{C}^N), \\ \mathbf{R}(\omega) &= [\mathbf{r}_1(\omega), ..., \mathbf{r}_N(\omega)] (\in \mathbb{C}^{M\times N}), \\ \mathbf{r}_i(\omega) &= [R_{1i}, ..., R_{Mi}]^T (\in \mathbb{C}^M), \\ \mathbf{v}(\omega) &= [V_1(\omega), ..., V_M(\omega)]^T (\in \mathbb{C}^M). \end{aligned} \quad (8)$$

The superscript T denotes the transpose operator. In this formulation, $\mathbf{A}(\omega)$ is known as the matrix related to the all possible finite source directions.

3. A maximum *a posteriori* sound source localization

We explain the relation between the existing ML-SSL framework [1] and the proposed MAP-SSL algorithm in our problem formulation. As the ML-SSL framework in [1], the combined total noise is defined as

$$\mathbf{v}_i^c(\omega) = \mathbf{r}_i(\omega)S_i(\omega) + \mathbf{v}(\omega). \quad (9)$$

Thus, the complex Gaussian likelihood of received signals can be written as

$$p(\mathbf{x}(\omega)|\mathbf{a}_i(\omega), S_i(\omega), \mathbf{Q}_i^c(\omega)) = (\pi)^{-M}|\mathbf{Q}_i^c(\omega)|^{-1}exp(-J_i(\omega)) \quad (10)$$

where

$$J_i(\omega) = [\mathbf{x}(\omega) - \mathbf{a}_i(\omega)S_i(\omega)]^H \mathbf{Q}_i^c(\omega)^{-1}[\mathbf{x}(\omega) - \mathbf{a}_i(\omega)S_i(\omega)]$$

and the covariance matrix

$$\begin{aligned} \mathbf{Q}_i^c(\omega) &= E[\mathbf{r}_i(\omega)S_i(\omega)(\mathbf{r}_i(\omega)S_i(\omega))^H] \\ &+ E[\mathbf{v}(\omega)\mathbf{v}^H(\omega)]. \end{aligned}$$

The superscript H denotes the Hermitian transpose operator. Here, reverberation and noise are assumed uncorrelated and follow the zero-mean complex Gaussian. The ML-SSL framework determines a direction of source as ith direction that leads to the largest value of (10) in all possible finite source directions. Considering all possible finite source directions, we formulate the joint complex Gaussian likelihood distribution of the received signals as

$$p(\mathbf{x}(\omega)|\mathbf{A}(\omega), \mathbf{s}(\omega), \mathbf{Q}^c(\omega)) = (\pi)^{-M}|\mathbf{Q}^c(\omega)|^{-1}exp(-J(\omega)) \quad (11)$$

where

$$J(\omega) = [\mathbf{x}(\omega) - \mathbf{A}(\omega)\mathbf{s}(\omega)]^H \mathbf{Q}^c(\omega)^{-1}[\mathbf{x}(\omega) - \mathbf{A}(\omega)\mathbf{s}(\omega)]$$

and the covariance matrix

$$\begin{aligned} \mathbf{Q}^c(\omega) &= E[\mathbf{R}(\omega)\mathbf{s}(\omega)(\mathbf{R}(\omega)\mathbf{s}(\omega))^H] \\ &+ E[\mathbf{v}(\omega)\mathbf{v}^H(\omega)]. \end{aligned}$$

To maximize (10) for the ith source, as in [1], we take the derivative of $J(\omega)$ with respect to $S_i(\omega)$, set it to zero and then have the following solution for $S_i(\omega)$,

$$S_i(\omega) = \frac{\mathbf{a}_i(\omega)^H \mathbf{Q}_i^c(\omega)^{-1}\mathbf{x}(\omega)}{\mathbf{a}_i(\omega)^H \mathbf{Q}_i^c(\omega)^{-1}\mathbf{a}_i(\omega)}. \quad (12)$$

The source vector of (11), $\mathbf{s}(\omega) = [S_1(\omega), ..., S_N(\omega)]^T$ is a sparse vector that has one non-zero element when an active source exists. Therefore, we obtain the source direction estimate by maximizing *a posteriori* of source with a sparse source prior. As the SBL in [7], the parametric form of the Gaussian source prior is defined as

$$p(\mathbf{s}(\omega); \boldsymbol{\gamma}) = \prod_{i=1}^{N} \mathcal{N}(S_i(\omega); 0, \gamma_i) \quad (13)$$

where $\boldsymbol{\gamma} = [\gamma_1, ..., \gamma_N]$ is a vector of source variances. Combing the likelihood and the source prior, the marginal likelihood is given as,

$$\begin{aligned} &p(\mathbf{x}(\omega); \boldsymbol{\gamma}, \mathbf{A}(\omega), \mathbf{Q}^c(\omega)) \\ &= \int p(\mathbf{x}(\omega)|\mathbf{A}(\omega), \mathbf{s}(\omega), \mathbf{Q}^c(\omega))p(\mathbf{s}(\omega); \boldsymbol{\gamma})d\mathbf{s}(\omega). \end{aligned} \quad (14)$$

We have our criterion that maximizes the marginal likelihood of (14), considering the sparse source prior. The source variances are estimated such that the marginal likelihood is maximized as shown below

$$\boldsymbol{\gamma}^* = \arg\max E_{p(\mathbf{s}(\omega);\boldsymbol{\gamma})}[p(\mathbf{x}(\omega)|\mathbf{A}(\omega), \mathbf{s}(\omega), \mathbf{Q}^c(\omega))]. \quad (15)$$

This is equivalent to minimizing the negative log-marginal likelihood. That is,

$$\boldsymbol{\gamma}^* = \arg\min_{\boldsymbol{\gamma}} \mathcal{L}(\boldsymbol{\gamma}) \quad (16)$$

with the negative log-marginal likelihood given by

$$\begin{aligned} \mathcal{L}(\boldsymbol{\gamma}) &\triangleq -\log \int p(\mathbf{x}(\omega)|\mathbf{A}(\omega), \mathbf{s}(\omega), \mathbf{Q}^c(\omega))P(\mathbf{s}(\omega); \boldsymbol{\gamma})d\mathbf{s}(\omega) \\ &= -\log p(\mathbf{x}(\omega); \boldsymbol{\gamma}, \mathbf{A}(\omega), \mathbf{Q}^c(\omega)) \\ &= \log|\Sigma_t(\omega)| + \mathbf{x}(\omega)^H \Sigma_t^{-1}(\omega)\mathbf{x}(\omega) \end{aligned} \quad (17)$$

where $\Sigma_t(\omega) \triangleq \mathbf{Q}^c(\omega) + \mathbf{A}(\omega)\Gamma\mathbf{A}^H(\omega)$ and $\Gamma = diag(\gamma_1, ..., \gamma_N)$. We use a fixed point update rule to estimate $\boldsymbol{\gamma}^*$ to converge fast, as in [8]. Given $\boldsymbol{\gamma}$, the posterior distribution of $\mathbf{s}(\omega)$ is given as

$$\mathbf{s}(\omega) \sim \mathcal{N}(\boldsymbol{\mu}(\omega), \Sigma(\omega)) \tag{18}$$

where

$$\begin{aligned} \boldsymbol{\mu}(\omega) &= \Gamma\mathbf{A}(\omega)^H\Sigma_t^{-1}(\omega)\mathbf{x}(\omega), \\ \Sigma(\omega) &= \Gamma - \Gamma\mathbf{A}(\omega)^H\Sigma_t^{-1}(\omega)\mathbf{A}(\omega)\Gamma. \end{aligned}$$

Finally, for a given $\boldsymbol{\gamma}^*$, we choose $\hat{\mathbf{s}}(\omega)$ as,

$$\hat{\mathbf{s}}(\omega) = \boldsymbol{\mu}^*(\omega) \tag{19}$$

where $\hat{\mathbf{s}}(\omega) = [\hat{S}_1(\omega), ..., \hat{S}_N(\omega)]^T$. The estimated source vector $\hat{\mathbf{s}}(\omega)$ has the one largest value that is related to the direction of source. The covariance matrix $\mathbf{Q}^c(\omega)$ is involved to consider the effect of reverberation in the proposed MAP-SSL algorithm. Note that the covariance matrix $\mathbf{Q}^c(\omega)$ determines the weight (significance) placed on the likelihood over the sparsity when obtaining the source estimate. As in [1], assuming all the reverberations and noises are uncorrelated, the covariance matrix is given as,

$$\mathbf{Q}^c(\omega) = diag(\kappa_1(\omega), ..., \kappa_M(\omega)) \tag{20}$$

where

$$\kappa_i(\omega) = E[|R_{i1}(\omega)|^2|S_1(\omega)|^2] + E[|R_{i2}(\omega)|^2|S_2(\omega)|^2] + \cdots + E[|R_{iN}(\omega)|^2|S_N(\omega)|^2] + E[|V_i(\omega)|^2].$$

But the diagonal element of covariance matrix, $\kappa_i(\omega)$ can be approximated for an active source as in [1] to

$$\kappa_i(\omega) = \beta|X_i(\omega)|^2 + (1-\beta)E[|V_i(\omega)|^2] \tag{21}$$

where β is a free parameter related to the power of the reverberant signal. Finally, the direction of source is estimated as the following criterion,

$$\hat{i} = \arg\max_{i\in 1,,,N} \int_\omega |\hat{S}_i(\omega)|d\omega \tag{22}$$

Comparing to the solution of (12), the proposed MAP-SSL algorithm has a sparse solution, enforcing the sources for the other directions to be near zeros in the all possible finite source directions.

4. Performance evaluation

To evaluate the performance of the proposed MAP-SSL algorithm, the experiments are performed on both synthetic and real data. The proposed MAP-SSL algorithm is also compared to the variants of ML-SSL framework in [1] for localizing a source in reverberant and noisy conditions.

4.1. The experiments on synthetic data

4.1.1. Acoustic environment

An acoustic room with size 6× 4 ×3 meters is modeled. Four omnidirectional microphones are placed linearly and near the center of room. The locations of microphones are (3, 1.13, 2), (3, 1.26, 2), (3, 1.39, 2) and (3, 1.52, 2), spacing 0.13m between microphones. A speaker is talking at a distance of about 2 m from microphones. Room impulse responses from a location of single speech to microphones for several locations of speaker are generated by image method [9]. To assess the impact of reverberation on source localization performance, we synthesize room impulse response with 300ms and 500ms reverberation time. The received signal sampled at 16kHz is obtained by convolving a speech with each room impulse response, adding white Gaussian noise. The Fourier transform of the received signals employs 32ms window, overlapping 16ms.

4.1.2. Construction of $\mathbf{A}(\omega)$ related to all possible finite source directions

The proposed MAP-SSL algorithm constructs $\mathbf{A}(\omega)$ related with the arrangement of microphones, assuming that a source is very far from the microphones. Considering the distance d_m(=0.13m $\times(m-1)$) between the 1st microphone and the mth microphone, the maximum possible time delay, $\tau_m^{'max}$ is calculated by,

$$\tau_m^{'max} = \lfloor \frac{d_m f_s}{C} \rfloor \tag{23}$$

where $\lfloor x \rfloor$ outputs the largest integer smaller than x, f_s is 16kHz, and C is 340(m/s). The range of the time delay, $\tau_m^{'}$ for the mth microphone, is from $-\tau_m^{'max}$ and $\tau_m^{'max}$. Therefore, the direction resolution of the source from (1) is about 15°. The dimension of the constructed $\mathbf{A}(\omega)$ is 4×12 matrix.

4.1.3. Experimental results of the SSL accuracy

To compare to the proposed MAP-SSL algorithm, the SRP-PHAT algorithm that is a version of the ML-SSL framework, and the ML-SSL framework perform hypothesis testing at about 15° intervals. Table 1 shows experimental results of source localization (SL) accuracy on the synthetic single source data according to reverberation and noise levels. We report the average accuracy, in terms of what portion of the SSL estimates (out of a total of 235 (time blocks) $\times$ 10 (various locations of the source)$\times$100 (monte carlo simulations)) is within 15° of the ground truth angles that we know. The parameter $\beta = 0.1$ of the ML-SSL framework is set to have best performance as in [1]. The

Table 1: Experimental results of the SSL accuracy on the synthetic single source. (A) reverberation time = 300 ms.(B) reverberation time = 500 ms.

Input SNR	SRP-PHAT	ML-SSL (β=0.1)	MAP-SSL (β=0.1)
20 dB	88.2 %	90.6 %	**91.7** %
15 dB	82.4 %	89.7 %	**91.5** %
10 dB	80.2 %	89.1 %	**91.4** %
5 dB	72.1 %	84.3 %	**90.6** %
0 dB	64.3 %	82.1 %	**90.1** %

(A)

Input SNR	SRP-PHAT	ML-SSL(β=0.1)	MAP-SSL (β=0.1)
20 dB	81.1 %	82.1 %	**90.4** %
15 dB	77.4 %	78.7 %	**89.3** %
10 dB	73.3 %	74.4 %	**88.9** %
5 dB	68.1 %	67.2 %	**87.2** %
0 dB	56.4 %	56.7 %	**86.3** %

(B)

MAP-SSL algorithm with $\beta = 0.1$ outperforms SRP-PHAT algorithm and ML-SSL framework for various reverberation and noise levels. Especially, in highly reverberant and noisy conditions, the improvement of the SSL accuracy is remarkable. The proposed MAP-SSL performs better than the SRP-PHAT algorithm and the ML-SSL framework in reverberation and noisy conditions.

4.2. The experiments on real data

We next test the proposed MAP-SSL algorithm on real data captured by 4 channel audio interface. Figure 1 represents an experimental setting in real room. The acoustic room size is $5.7\times 2\times 3$ meters. Four omnidirectional microphones are placed linearly and near the center of room. The locations of microphones are (3, 0.74, 2), (3, 0.87, 2), (3, 1, 2) and (3, 1.13, 2), spacing 0.13m between the microphones. A speaker is talking at a distance of about 1.5 m from microphones. The ground truth angles of the speaker are $[-90^\circ, -75^\circ, ..., 90^\circ]$ with 15° direction resolution. The Fourier transform of the received signals sampled at 16kHz employs 32ms window, overlapping 16ms. For each truth angle, the length of the received signal is about 3s. We report the results on the percentage of frames that are within 15° of the ground truth angles. Table 2 shows experimental results of the SSL accuracy on the real data. The proposed MAP-SSL algorithm performs better than the SRP-PHAT algorithm and the ML-SSL framework.

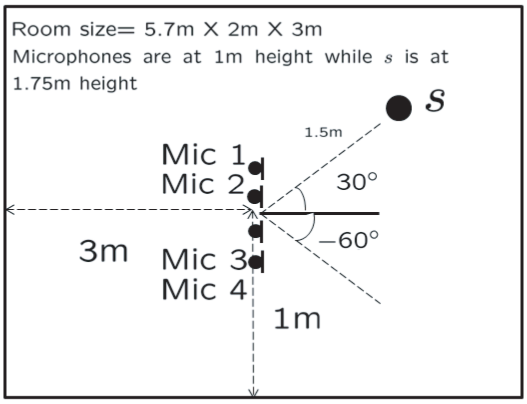

Figure 1: An experimental setting in real room.

Table 2: Experimental results of the SSL accuracy on the real data.

SRP-PHAT	ML-SSL (β=0.1)	MAP-SSL (β=0.1)
80.2 %	82.2 %	**85.4** %

4.3. The limitation of the proposed MAP-SSL algorithm

The proposed MAP-SSL algorithm has a limitation. If the matrix $\mathbf{A}(\omega)$ has a high overcompleteness ratio [7], i. e. that the number of all possible finite source directions to the number of the microphones ratio is very high, the performance of the MAP-SSL algorithm may be degraded due to not obtaining the exact sparse solution for $\mathbf{s}(\omega)$. But, if a proper overcompleteness ratio is used to construct the matrix $\mathbf{A}(\omega)$, the proposed MAP-SSL algorithm can get the accurate source location.

5. Conclusions

In this paper, a MAP-SSL algorithm is proposed. When a source is active, the criterion in deriving the proposed MAP-SSL algorithm is similar to the criterion used to derive the existing ML-SSL framework, except that in our criterion a sparse source prior that enforces a sparse solution is added. The sparse source prior plays a key role in improving the source localization performance over other algorithms such as the variants of ML-SSL framework. In experiments, the proposed MAP-SSL algorithm outperforms the variants of the ML-SSL framework for both the synthetic data that have various reverberation and noise levels and the real data.

6. Acknowledgements

This work was supported (National Robotics Research Center for Robot Intelligence Technology, KAIST) by Ministry of Knowledge Economy under Human Resources Development Program for Convergence Robot Specialists.

7. References

[1] C. Zhang, D. Florencio, D. E. Ba, and Z. Zhang, “Maximum likelihood sound source localization and beamforming for directional microphone arrays in distributed meetings,” *IEEE Transactions on Multimedia*, vol. 10, no. 4, pp. 538–548, 2008.

[2] B. D. Rao T. Gustafsson and M. Trivedi, “Source localization in reverberant environments: modeling and statistical analysis,” *IEEE Transactions on Speech and Audio Processing*, vol. 11, no. 6, pp. 791–803, 2003.

[3] Y. Rui and D. Florencio, “Time delay estimation in the presence of correlated noise and reverberation,” in *Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing*, 2004.

[4] B. Champagne, S. Bedard, A. Stephenne, I. Telecommun, and Q. Verdun, “Performance of time delay estimation in the presence of roomreverberation,” *IEEE Transactions on Speech and Audio Processing*, vol. 4, no. 2, pp. 148–152, 1996.

[5] H. Silverman J. DiBiase and M. Brandstein, *Microphone Arrays*, Springer, 2001.

[6] D. Malioutov, M. Cetin, and A. S. Willsky, “A sparse signal reconstruction perspective for source localization with sensor arrays,” *IEEE Transactions on Signal Processing*, vol. 53, no. 8, pp. 3010–3022, 2005.

[7] D.P. Wipf and B.D. Rao, “Sparse Bayesian learning for basis selection,” *IEEE Transactions on Signal Processing*, vol. 52, no. 8, pp. 2153–2164, 2004.

[8] D.P. Wipf and B.D. Rao, “An empirical Bayesian strategy for solving the simultaneous sparse approximation problem,” *IEEE Transactions on Signal Processing*, vol. 55, no. 7, pp. 3704–3716, 2007.

[9] J.B. Allen and D.A. Berkley, “Image method for efficiently simulating small-room acoustics,” *Journal of the Acoustical Society of America*, vol. 65, no. 4, pp. 943–950, 1979.